

Supplementary Materials

Network-theoretic modeling of DIEP flap perfusion redundancy for computational microsurgical planning

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Analytical specification and results

1. Evidence basis and reference verification

We used a narrative anatomical and methodological evidence assessment to identify pathways and computational methods relevant to DIEP vascular redundancy. Sources were selected for their contribution to model construction: anatomical studies described perfusion territories or deep/superficial venous pathways; flow studies addressed quantitative DIEP supply; mathematical sources defined methods implemented in the analysis. This assessment was purpose-specific and non-systematic.

Targeted title, DOI and topic searches located primary bibliographic, publisher and institutional records. The final editorial update added recent clinical studies of perforator geometry, quantitative regional perfusion, superficial venous drainage and CTA-based perforator mapping. Representative searches included “DIEP perfusion area versus volume”, “DIEP second venous drainage indocyanine green”, “DIEP perfusion mapping laser speckle”, “superficial deep venous system CTA”, “automated CTA perforator mapping”, “Kron reduction Dörfler Bullo” and the titles of the original mathematical method papers. Sources were retained for their direct support of the cited anatomical, imaging or computational statement; this was not a systematic literature review.

Each reference includes a verified DOI. The anatomical evidence informed pathway types; numerical weights and conductance budgets were defined synthetically. Published findings from Sowa *et al.* provided clinical context, without reanalysis of their patient data. The assessment provides a focused evidence basis without systematic-review screening counts or a claim of exhaustive novelty verification. The main-text bibliography contains 19 references, of which 10 (52.6%) were published in 2023–2026; the 2021 article is not needed to meet the five-year recency requirement. The author names for Suhail *et al.* follow the published author-name correction (DOI:10.1016/j.jprra.2025.07.002).

2. Complete model definitions

2.1 Original nine-node models

The original graphs have the fixed node order DIEA, P1, P2, Zone I, Zone II, Plexus, DIEV, left SIEV and right SIEV. Supplementary Table 1 combines their edge lists; a zero denotes an absent edge. All weights are dimensionless. The original SDC joins left SIEV to DIEV and is therefore ipsilateral in this labeling. The compartmental analysis separately tests ipsilateral and contralateral SDC placement. The original low-weight second-perforator connections are numerical surrogates used in the original connectivity benchmark. The compartmental baseline instead removes the direct second-perforator pedicle connections.

Supplementary Table 1. Original edge specifications

Edge	A weight	B weight
DIEA-P1	1.00	1.00
DIEA-P2	1.00	0.03
P1-P2	0.30	0.00
P1-ZoneI	0.65	0.65
P2-ZoneII	0.65	0.03
P1-Plexus	0.45	0.45
P2-Plexus	0.45	0.03
ZoneI-Plexus	0.30	0.30
ZoneII-Plexus	0.30	0.03
ZoneI-leftSIEV	0.55	0.55
ZoneII-rightSIEV	0.55	0.55
leftSIEV-rightSIEV	0.45	0.00
P1-DIEV	0.55	0.55
P2-DIEV	0.55	0.03
leftSIEV-DIEV	0.40	0.00

Zeros indicate absent edges. Original identifiers are preserved for exact reproduction; leftSIEV and rightSIEV refer to ipsilateral and contralateral superficial collectors in the original labeling.

2.2 Compartment nodes, positions and resistance budgets

For n tissue units, the nodes are AS (arterial source), DIEA, P1a, P2a, P1v, P2v, DIEV, S_L, S_R and VD (deep recipient outlet), plus a_i and v_i for $i = 0, \dots, n - 1$. VL and/or VR are added only when an ipsilateral and/or contralateral superficial recipient outlet is active. There are 18, 26 or 42 nodes for deep-only models with 4, 8 or 16 units. The eight-unit reference enhanced graph has 61 edges; its baseline has 56.

The geometry uses two rows with $y = -0.35$ and $+0.35$ and $n/2$ evenly spaced x coordinates from -1 to $+1$. Units with $x < 0$ are ipsilateral; units with $x > 0$ are contralateral. The illustrative perforator positions are $(-0.5, -0.15)$ and $(+0.5, -0.15)$. Within each side, branch allocation fractions h_i are proportional to $\exp[-2((x_i - x_p)^2 + (y_i + 0.15)^2)]$ and normalized to sum to 1. The coordinates define normalized illustrative positions.

The fixed conductance budgets are: AS-DIEA, 2.0; DIEA-P1a, 1.0; P1v-DIEV, 0.55; DIEV-VD, 1.5; arterial collector-to-tissue branches, $0.65h_i$ per side; tissue-to-deep collector branches, $0.55h_i$ per side; tissue-to-superficial collector branches, $0.55h_i$ per side; and a_i - v_i , $1/n$ for every tissue unit. Thus, the total tissue-exchange conductance is always 1. A supplementary superficial recipient connection, when specified, has conductance 0.8. Non-anastomosed superficial collectors are never assigned an outlet pressure.

For the five enhancement levels (p, m, s, u, l), each ranging from 0 to 1, the variable conductances are DIEA-P2a = p, P2v-DIEV = $0.55p$, S_L-S_R = $0.45m$, the selected SDC = $0.40s$ and P1a-P2a = $0.30l$. P2's intratissue collectors and branches remain represented when its direct pedicle connections are absent, permitting lateral supply and drainage through tissue networks. The enhancement therefore recruits a direct perforator supply/drainage pair.

Local collaterals connect horizontally adjacent units in the same row and vertically aligned units across rows. Distributed collaterals connect every pair of tissue units. Each candidate receives weight proportional to inverse Euclidean distance; weights are normalized across all candidate pairs. The total conductance budget is $0.12 + 0.60u$ separately in the arterial and venous layers. Both baseline layers therefore retain positive collateral conductance. The plexus term represents an effective distributed pathway budget.

All source pressure values are 1 and all active recipient pressures are 0. Internal pressures are solved by linear conservation. Pressure-determined direction may differ between configurations; lateral venous connections are not modeled as fixed one-way valves. Tissue exchange is the only connection between arterial and venous layers. The code exports the reference graph, pressures and directed edge flows in compartment_A.json and compartment_B.json.

2.3 Scenario construction and interpretation

The Cartesian product of $n = \{4,8,16\}$, collateral pattern = {local,distributed}, SDC side = {ipsilateral,contralateral} and outlet arrangement = {deep,deep + ipsilateral,deep + contralateral,deep + bilateral} defines 48 formulations. Each contains 32 enhancement coalitions, yielding 1,536 intact hydraulic solutions. A and B in the compartmental output filenames mean enhanced and baseline endpoints; they identify the compartmental states independently of the original nine-node archetypes. Supplementary Table 2 summarizes all formulation-level endpoint ratios. Full-precision absolute values and regional Shapley contributions are supplied as CSV files.

Supplementary Table 2. All 48 structural formulations

Tissue units	Collaterals	SDC side	Outlets*	Total Q ratio	Contra Q ratio
4	local	ipsi	Deep	1.992	33.011
4	local	ipsi	+ ipsi	1.693	27.541
4	local	ipsi	+ contra	2.076	8.147
4	local	ipsi	+ both	1.735	9.805
4	local	contra	Deep	1.992	35.003
4	local	contra	+ ipsi	1.727	28.740
4	local	contra	+ contra	2.036	8.161
4	local	contra	+ both	1.735	9.845
4	distributed	ipsi	Deep	1.986	21.824
4	distributed	ipsi	+ ipsi	1.687	18.223

Tissue units	Collaterals	SDC side	Outlets*	Total Q ratio	Contra Q ratio
4	distributed	ipsi	+ contra	1.984	5.892
4	distributed	ipsi	+ both	1.697	6.989
4	distributed	contra	Deep	1.986	23.028
4	distributed	contra	+ ipsi	1.720	18.969
4	distributed	contra	+ contra	1.946	5.894
4	distributed	contra	+ both	1.697	7.015
8	local	ipsi	Deep	1.994	38.070
8	local	ipsi	+ ipsi	1.696	31.783
8	local	ipsi	+ contra	2.123	9.881
8	local	ipsi	+ both	1.754	11.798
8	local	contra	Deep	1.994	40.422
8	local	contra	+ ipsi	1.730	33.191
8	local	contra	+ contra	2.082	9.902
8	local	contra	+ both	1.754	11.847
8	distributed	ipsi	Deep	1.986	19.591
8	distributed	ipsi	+ ipsi	1.688	16.382
8	distributed	ipsi	+ contra	1.963	5.495
8	distributed	ipsi	+ both	1.689	6.474
8	distributed	contra	Deep	1.986	20.636
8	distributed	contra	+ ipsi	1.721	17.038
8	distributed	contra	+ contra	1.926	5.494
8	distributed	contra	+ both	1.689	6.497
16	local	ipsi	Deep	1.998	76.159
16	local	ipsi	+ ipsi	1.701	63.526
16	local	ipsi	+ contra	2.222	17.299
16	local	ipsi	+ both	1.792	21.164

Tissue units	Collaterals	SDC side	Outlets*	Total Q ratio	Contra Q ratio
16	local	contra	Deep	1.998	81.162
16	local	contra	+ ipsi	1.736	66.484
16	local	contra	+ contra	2.177	17.350
16	local	contra	+ both	1.792	21.256
16	distributed	ipsi	Deep	1.988	23.215
16	distributed	ipsi	+ ipsi	1.690	19.398
16	distributed	ipsi	+ contra	2.000	6.208
16	distributed	ipsi	+ both	1.705	7.373
16	distributed	contra	Deep	1.988	24.511
16	distributed	contra	+ ipsi	1.723	20.199
16	distributed	contra	+ contra	1.962	6.211
16	distributed	contra	+ both	1.705	7.400

*All configurations retain the deep outlet. “+ ipsi”, “+ contra” and “+ both” add corresponding superficial recipient connections. Ratios compare enhanced with baseline models under the same formulation; they are not clinical effect estimates.

3. Feature attribution and graph-resolution benchmark

Exact Shapley attribution uses the 32 coalition values for each specified value function. The weight for a coalition of size k that excludes the selected feature is $k!(4 - k)!/5!$. The original-graph enhancement definitions are precisely the B-to-A edge changes: second perforator changes four edges; MCLV adds one edge; SDC adds one edge; the plexus bridge increases Zone II–Plexus from 0.03 to 0.30; the linking feature adds P1-P2. In the compartmental game, the five players control the continuous enhancement parameters in Section 2.2. Both games allocate a defined model-output change.

The subdivision benchmark replaces each edge of conductance g by k serial edges of conductance kg . The resulting networks are hydraulically equivalent at the original nine nodes. Eliminating only the introduced internal nodes by the Schur complement recovers the original

Laplacian. This provides an exact-resistance benchmark for representation dependence. Supplementary Table 3 gives all tested subdivisions.

Supplementary Table 3. Conductance-preserving subdivision and reduction

Graph	Segments/edge	Nodes	Raw λ_2	Reduced λ_2
A	1	9	0.638034	0.638034
A	2	24	0.297102	0.638034
A	4	54	0.141661	0.638034
A	8	114	0.068918	0.638034
B	1	9	0.033342	0.033342
B	2	21	0.018399	0.033342
B	4	45	0.009376	0.033342
B	8	93	0.004696	0.033342

Reduction retains the same original nine nodes. Added segment conductance equals subdivision count times original conductance.

4. Exact degradation and single-segment contingencies

For each original capacity graph, the source is DIEA and the sink is DIEV. Each undirected weighted edge is interpreted as two opposing capacity arcs for this diagnostic only. For every surviving-edge subset, maximum flow equals the smallest source–sink cut capacity. With nine nodes, there are $2^7 = 128$ distinct source-side assignments of the nonterminal nodes. The analysis enumerates $2^{15} = 32,768$ subsets for A and $2^{12} = 4,096$ for B. Original weights are multiplied by 100 to compute the cut sums as integers, then rescaled.

For k removed edges from m total edges, every subset of size $m - k$ has equal probability under uniformly random-order deletion. Averaging over these subsets therefore gives the exact expected normalized capacity at k/m without Monte Carlo error. Supplementary Table 4 reports the discrete means and population standard deviations. Linear interpolation at 0.5 gives $f_{50} = 0.322400187182$ for A and 0.287457191781 for B. The fractions describe removal of edges in the original capacity benchmark.

Supplementary Table 4. Exact original-graph capacity-degradation curves

Graph	Failed edges	Failed fraction	Mean residual	Population SD
A	0	0.000000	1.000000	0.000000
A	1	0.066667	0.888889	0.158543
A	2	0.133333	0.785397	0.196203
A	3	0.200000	0.687326	0.218023
A	4	0.266667	0.587253	0.236953
A	5	0.333333	0.482884	0.248810
A	6	0.400000	0.376976	0.248351
A	7	0.466667	0.277819	0.235512
A	8	0.533333	0.194074	0.214118
A	9	0.600000	0.129004	0.185907
A	10	0.666667	0.080608	0.154043
A	11	0.733333	0.045763	0.120463
A	12	0.800000	0.021832	0.085917
A	13	0.866667	0.006984	0.050121
A	14	0.933333	0.000000	0.000000
A	15	1.000000	0.000000	0.000000
B	0	0.000000	1.000000	0.000000
B	1	0.083333	0.837644	0.351759
B	2	0.166667	0.689655	0.439571
B	3	0.250000	0.554859	0.469294
B	4	0.333333	0.432811	0.465138
B	5	0.416667	0.324190	0.438280
B	6	0.500000	0.230408	0.394809
B	7	0.583333	0.152625	0.338422
B	8	0.666667	0.091118	0.271841
B	9	0.750000	0.045455	0.197305
B	10	0.833333	0.015152	0.115912

Graph	Failed edges	Failed fraction	Mean residual	Population SD
B	11	0.916667	0.000000	0.000000
B	12	1.000000	0.000000	0.000000

Every surviving-edge subset is equally weighted at each failure count. Residual capacity is divided by the intact maximum-flow capacity. SD denotes standard deviation across subsets, not uncertainty in the exact mean.

The compartmental contingency analysis removes one modeled segment at a time and recomputes the pressure solution. All reference edges are included, including common arterial and recipient venous connections. A disconnected component with no pressure boundary is assigned an arbitrary zero reference pressure and carries no flow. Numerical values near zero after terminal failure are interpreted within a 10^{-9} tolerance. Complete edge-specific results are in `single_edge_failures.csv`.

5. Perturbations, numerical verification and reproduction

Original spectral sensitivity uses 2,000 independent draws per noise level $\sigma = 0.10, 0.20, 0.30$ or 0.50 , with seeds 4200, 4201, 4202 or 4203, respectively. Each edge is multiplied by $\exp(\sigma Z)$, $Z \sim N(0,1)$, independently across edges and original archetypes. At $\sigma = 0.20$, a multiplier's approximate central 95% interval is 0.676–1.480; the distribution is not bounded by these values or by $\pm 20\%$.

Compartmental sensitivity uses 1,000 paired draws at $\sigma = 0.20$ with seed 8400. The same multiplier is used for an edge present in both endpoint models. The continuous response surface instead uses deterministic 21-point grids from 0 to 1 for MCLV and SDC under deep-only and additional contralateral superficial drainage. No random sampling is involved in the structural ensemble, continuous grids, Shapley calculations or exact capacity-degradation curves.

Verification checks source inflow against summed tissue exchange, internal-node balance, positivity of intact tissue exchange, Shapley efficiency and recovery of the original Laplacian after serial subdivision and reduction. The reference pressure solution also passes the energy identity $p^T L p = Q$ for unit pressure difference and agrees with its reduced boundary flow.

Independent checks of the enumerated minimum-cut method against SciPy's maximum-flow solver agreed for 50 edge subsets per original archetype. These checks establish computational consistency.

The computational environment was Python 3.12.13, NumPy 2.3.5, SciPy 1.17.0 and Matplotlib 3.10.8. From the extracted supplement directory, run `python code/revise_analysis.py`, `python code/budget_sensitivity.py`, `python code/verify_analysis.py` and `python code/make_figures.py`. These regenerate the reference analyses, conductance-budget contrasts, independent flow checks and computational figure components, respectively; subsequent Illustrator presentation edits are described in Section S6. Fixed seeds reproduce the random-number sequences within the stated environment. Minor floating-point differences may occur across numerical library implementations. All reported model weights and graph-construction rules are in the code; no unavailable external dataset or paid service is needed to reproduce the calculations.

6. Figure and graphical abstract provenance

AI-assisted preparation covered Figures 1-4, Supplementary Figures 1 and 2, and the graphical abstract. Assistance included creation of individual figure components and plotting code. The authors assembled and edited components in Adobe Illustrator 2026 (version 30.1, the release line available in February 2026). The supplied Matplotlib 3.10.8 scripts generate the computational plots and graph drawings from the documented model definitions and numerical results; they do not reproduce every subsequent Illustrator layout or presentation edit. Figure 1A is an explanatory schematic rather than patient-derived anatomy. Figure 1C and the graphical abstract represent the same eight-unit reference graph. All plotted values, network connections and numerical annotations must be read as outputs of the stated computational model, not as observations generated or inferred by an image-generation system.

The AI tools and available version information are disclosed in the main manuscript. The authors reviewed the final figures against the supplied model definitions and numerical outputs. Exact web-service session/build identifiers were not documented in the available records. The previously stated exclusively programmatic provenance has been corrected to reflect the combined AI-assisted and author-directed Illustrator workflow.

7. Conductance-budget sensitivity

For each of the 48 structural formulations, both second-perforator pedicle conductances were multiplied by $k = 0.10, 0.25, 0.50, 1, 2$ or 4 . Thus, their enhanced conductances were k and $0.55k$, respectively; the baseline remained unchanged. Each multiplier defines a complete 32-coalition Shapley calculation. The additional equal-increment control assigned a summed edge-conductance increase of 0.40 to each feature. Relative to the reference increments of $1.55, 0.45, 0.40, 1.20$ and 0.30 , the corresponding scale factors were $0.40/1.55, 0.40/0.45, 1, 0.40/1.20$ and $0.40/0.30$. The baseline collateral budget remained 0.12 in each layer; only the 0.60 enhancement per layer was rescaled.

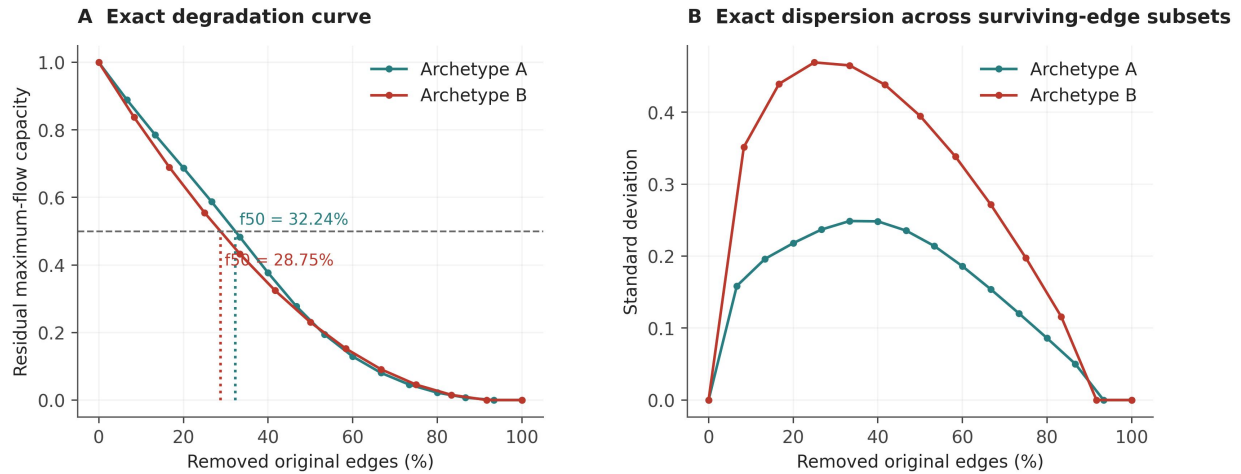
The seven budget contrasts contain 336 formulation–budget combinations and 10,752 coalition solutions, including replication of all 1,536 reference-budget solutions. The unit-multiplier Shapley values reproduced the original structural analysis exactly. Maximum errors were 2.66×10^{-15} for internal-node conservation, 5.55×10^{-17} for Shapley efficiency and 1.44×10^{-15} for the equal-increment check; all intact tissue-exchange flows were positive. Full-precision outputs are provided in `budget_sensitivity.csv`, `budget_summary.csv` and `budget_validation.json`; `code/budget_sensitivity.py` specifies the calculation.

The second perforator ranked first in all formulations at all tested values of k from 0.25 to 4 and under equal increments. At $k = 0.10$, the second perforator and arterial linking vessel each ranked first in 18 formulations, and SDC in 12. These stress tests distinguish effects of aggregate budget from pathway placement within this model family. They do not define an empirically plausible conductance range or equalize physiological function across different pathways.

Supplementary Table 5. Leading feature under conductance-budget contrasts

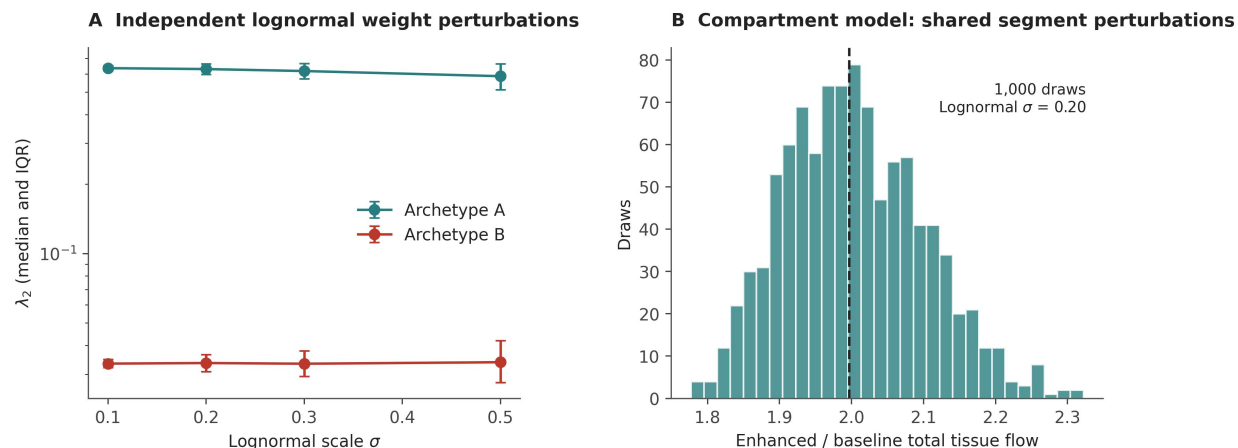
Budget contrast	Second perforator	SDC	Arterial link	Total Q ratio range
Perforator $\times 0.10$	18	12	18	1.366–1.816
Perforator $\times 0.25$	48	0	0	1.469–1.950
Perforator $\times 0.50$	48	0	0	1.575–2.085
Perforator $\times 1$	48	0	0	1.687–2.222
Perforator $\times 2$	48	0	0	1.775–2.334
Perforator $\times 4$	48	0	0	1.834–2.410
Equal increments	48	0	0	1.468–1.960

Each row contains 48 formulations. Counts identify the largest total-throughput Shapley contributor; there were no ties, and neither MCLV nor plexus enhancement ranked first. Perforator multipliers scale its arterial and venous pedicle conductances together. Equal increments assign each feature a summed edge-conductance increase of 0.40. Ratios compare enhanced with baseline throughput within each formulation.



Supplementary Figure 1. Exact degradation of the original capacity graphs. (A) Mean normalized maximum-flow capacity at each failed-edge fraction, with f50 located by linear interpolation. (B) Population standard deviation across all surviving-edge subsets at each step. A includes 32,768 subsets and B 4,096. These are deterministic enumerations, not confidence intervals or tissue-perfusion predictions. B has greater dispersion through the early and middle

degradation steps. Preparation included AI assistance; author assembly and editing are described in Section 6.



Supplementary Figure 2. Multiplicative lognormal weight perturbations. (A) Original λ_2 medians and interquartile ranges from 2,000 independent draws per σ and archetype, with seeds 4200-4203. The y-axis is logarithmic; (B) Enhanced/baseline total-throughput ratios in the eight-unit reference compartmental model for 1,000 paired draws, $\sigma = 0.20$, seed 8400. Shared segments receive the same multiplier. Factors $\exp(\sigma Z)$ are positive and unbounded; $\sigma = 0.20$ does not mean $\pm 20\%$. Distributions describe selected modeling perturbations, not patient uncertainty. Preparation included AI assistance; author assembly and editing are described in Section 6.